REMARKS/ARGUMENTS

Reconsideration of the subject application is requested. it is believed that the application, as now amended, should be allowed because of the following reasons.

The U.S. Supreme Court and Federal Circuit have recognized that in a crowded field, such as the field of pick-up truck cargo box rails, small differences distinguish an invention from the prior art.

The specification has been amended to correct the error which the Examiner noted. Claim 14 has been cancelled to overcome the Examiner's double patenting objection. Independent claim 11 has been cancelled and replaced by new claim 23 which is believed to be patentably distinct from the Examiner's references. Independent claims 6 and 22 have been amended to overcome the Examiner's 102(b) rejections.

Johnson (4,650,382) discloses an "L" shaped pick-up truck rail having a small diameter uppermost rod portion. The "L" shape which is a dominant feature of the patent is formed by an elongated bottom flange 21 and a coextensive elongated upright side flange 22 (see Johnson, column 2, lines 3 - 5). Not only does the Johnson rail differ substantially in appearance to Applicant's pick-up truck rail, it is also substantially inferior in torsional and bending stiffness and strength.

As pointed out in Seely and Smith 2nd edition, <u>Advanced Strength of Materials</u>, structural members such as I beams, channels, etc. are used to resist bending loads. Frequently these members are subjected to torsion, in addition to bending. Even though a torsional moment in these members may be very small, the lack of torsional strength and stiffness of such members makes their torsional performance of importance (see

attachment).

Seely, page 274, also points out that "(b) The torsional stiffness of a member with a long, straight, thin rectangular cross section is approximately the same as that of a member having an L-shaped or U-shaped section, etc., provided that the width of the section and the length of the median line remains constant."

From Seely, it is obvious that neither Johnson, 4,650,382 (slender "L" or "V" member); Elwell, 6,146,069 (slender "U" member) and Anderson, 2002/0012576 provide for loads with torsional components and will greatly deflect when subjected to torsion.

On the other hand, the present invention provides the superb torsional stiffness of a large diameter thin wall tube along with the simplicity of an integral "L" shaped attachment and is fifty times stiffer in torsion than Johnson or Elwell. The torsional stiffness reinforces the cargo box side panels and spreads torsional loads along the cargo box upper rails.

Johnson, 4,650,382 teaches a very slender rod shaped, rather than tubular upper portion, having a diameter which is 3 -5 times the thickness of a support flange. With this restriction, the diameter of the rod shaped portion with a support flange having a typical thickness in the order of 0.1" thick, is only 0.3 to 0.5 inches.

Amended independent claims 6 and 22 have been amended and new claim 23 particularly point out that the claimed inventions are directed to tubular pick-up truck rails having dominant upper tubular portions. They are further patentably distinguishable under 35 USC 102(b) and 35 USC 103(a) by the positive limitation of rails with tubular shaped upper portions for effectively resisting torsional and bending stresses. Since the remaining

claims depend from these claims, for the same reason they are patentably distinguisable from the applied references.

Amended claim 24 which depends from claim 23 includes the additional positive limitation of an upper tubular rail portion having a diameter which is substantially more than three to four times the thickness of the vertical downward extending planar wall portion.

Amended claim 25 which depends from claim 23 includes the additional positive limitations of an upper tubular rail portion having a wall thickness which is about equal to the thickness of the vertical downward extending planar wall portion and a diameter which is substantially more than three to four times the thickness of the adjoining vertical downward extending planar wall portion. The remaining dependent claims include limitations which further distinguish Applicant's invention from the references.

Amended claim 26 which depends from claim 23 includes the additional positive limitation of an upper tubular rail portion having a diameter of ten to thirty times the thickness of the adjoining vertical downward extending planar wall portion.

In view of the above, its is contended that the claims as now presented are directed to combinations which are neither anticipated nor obvious over the references and it is requested that this application be passed for allowance.

Respectfully submitted,

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Advanced Mechanics of materials
Seely and Smith
2nd edition

Chapter 9

TORSIONAL RESISTANCE OF BARS HAVING NON-CIRCULAR CROSS SECTIONS

84 Introduction. If a bar having a constant circular cross section of diameter d is subjected to twisting couples T at its ends, the shearing stress at any point on a transverse cross section varies directly as the distance of the point from the center of the section (see Art. 11) and is expressed by the equation

$$\tau = Tc/J \tag{287}$$

in which c is the distance to the point, and $J = \pi d^4/32$ is the polar moment of inertia of the cross section. Furthermore, in accordance with Art. 17, a shearing stress of equal magnitude acts at the same point on a longitudinal plane. It should be recalled also that the angle of twist θ of the cylindrical bar is

$$\theta = Tl/JG \qquad \text{or} \qquad \phi = \theta/l = T/JG \tag{288}$$

in which l is the length of the bar, G is the shearing modulus of elasticity of the material, and ϕ is the angle of twist per unit of length.

Equations 287 and 288 apply only to bars or shafts having constant circular cross sections, for which plane cross sections remain plane as the bar is twisted, and are valid only when the shearing stress in the shaft does not exceed the shearing elastic strength of the material.

Non-circular Section. In 1853, St. Venant in his classical memoir on the torsion of prismatic bars showed that, if a bar whose transverse cross section is *not* circular is twisted by applying moments at its ends, a plane transverse section before twisting does *not* remain a plane section after twisting; it becomes a warped surface, and this warping is accompanied by an increase of shearing stress in some parts of the section and a decrease in other parts as compared with stresses that would occur if the section did not warp but remained a plane, as in a bar having a circular cross section.

For example, in a bar with an elliptical section the shearing stress has its maximum value at the ends of the minor axis, that is, at the points on the surface *nearest* to the axis of the bar; whereas, if the plane section remained plane so that the elastic shearing strain (and also stress) at any point were proportional to the distance of the point from the axis of twist, the strain and stress on the surface of the bar would have their *minimum* values at the end of the minor axis.

Similarly, the maximum shearing stress in a bar with a rectangular section is at the center of the long side, that is, at a point on the surface *nearest* to the axis of the bar; and the shearing stress at each corner of the section is zero. Furthermore, the polar moment of inertia of the cross-sectional area of a bar with a non-circular section has a very different effect on the torsional stiffness and strength of the bar than on those of a bar with a circular cross section. This fact is brought out by a statement often referred to as St. Venant's paradox; namely, if two solid bars of the same ideal elastic material have non-circular cross-sectional areas that are equal but different in shape, the one with the smaller polar moment of inertia has the greater torsional stiffness; it also has the greater strength, provided that the areas are everywhere convex (do not have re-entrant angles).

If a section has a sharp internal corner (re-entrant angle) such as that caused by a keyway in a shaft, the warping of a transverse section causes at the internal corner a shearing stress which theoretically is infinitely large.

As a rule, structural members such as I beams, channels, etc., are used to resist bending loads, but frequently these members are subjected to torsion in addition to bending, and, although the torsional moment may be relatively small, the lack of torsional strength and stiffness of such members makes their torsional behavior of importance.

The analysis given in the next article of the torsional stresses in a bar having a rectangular section is an approximate but very useful and relatively simple solution, due mainly to Bach.

85 Torsion of bar having a rectangular cross section. In Fig. 144*a* is shown a portion of a bar having a rectangular cross section, the bar being subjected to twisting moments T at its ends. In order to find the shearing stresses in the bar in terms of the twisting moment T and the dimensions b and h of the cross section, the procedure outlined in Art. 8, Chapter 2, is employed.

A plane is passed through the bar, cutting it in a direction perpendicular to the longitudinal axis (the axis of twist). The portion of the body lying on the left side of this plane is shown in Fig. 144*a*. The in-

ternal forces $\tau_x\,da$ and $\tau_y\,da$ are the components in the x and y directions, respectively, of the shearing force $\tau\,da$ on the area da. The equations of equilibrium require that the external twisting moment T shall be equal and opposite to the internal twisting moment, and hence

$$T = \int [y\tau_x\,da + x(-\tau_y)\,da] \tag{289}$$

in which x, y, τ_x, and τ_y must be given signs in accordance with the conventions shown in Fig. 144.

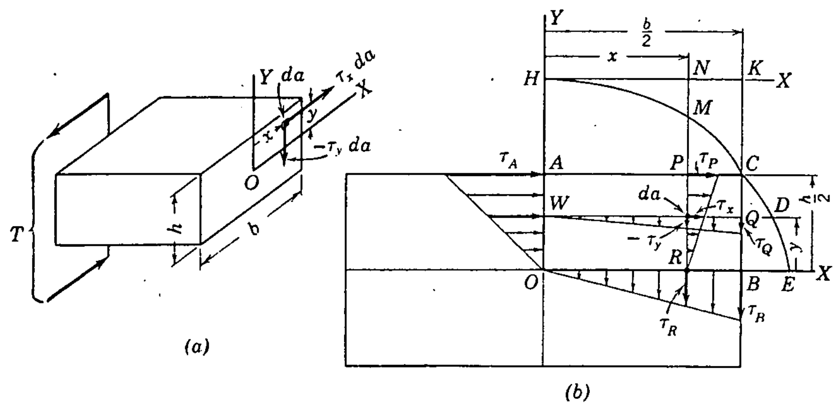


FIG. 144 Torsional shearing stresses in rectangular bar.

In order to integrate Eq. 289, the distribution of the stresses τ_x and τ_y must be obtained. In accordance with Art. 8, this distribution is obtained by making a study of the strains in the bar. When a bar having a rectangular section is twisted, it takes the form shown in Fig. 145*a*. The warping of the sections causes the shearing strain (and hence shearing stress) to be maximum at the center of the long side (at A, Figs. 145*a* and 144*b*) and to be zero at the corners, at C. These facts are obtained from a study of Figs. 145*a* and 145*b*, which show that the greatest distortion of the squares occurs at the middle of the long side and the least (zero) at the edge of the bar. Figure 145*c* shows the warping of the cross section. Points in the quadrants marked plus move in one direction longitudinally; those in quadrants marked minus move in the opposite direction. The fact that the shearing stress must be zero at the corner may also be shown as follows. The shearing stress on any transverse section at any point in the surface must be tangent to the surface; therefore, since C is a point on the side AC (Fig. 144*b*) and also on the side CB, the stress at C must be zero since it could not be tangent to both sides.

As shown in Fig. 145*a*, the strain (and hence stress) is assumed to vary from A to C (Fig. 144*b*) approximately as the ordinates of a parabola. Thus in Fig. 144*b* the stresses along AC are indicated by the ordinates to the parabola HMC having its vertex at H. In Fig. 144*b*, let τ_A represent the stress at A, and τ_P represent the stress at any point P on the

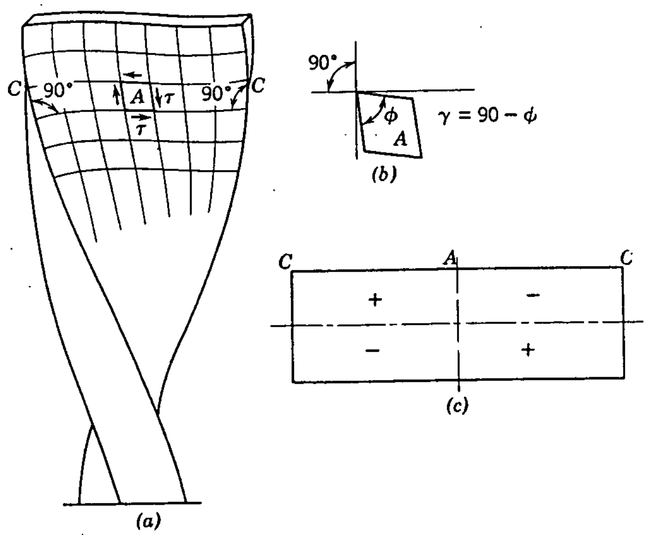


FIG. 145 Views showing warping of sections of bars having non-circular cross sections. (From Bach, reference No. 1.)

periphery between A and the corner C. If HMC is a parabola in which AH represents τ_A and PM represents τ_P, we have from the equation of a parabola

$$\frac{MN}{CK} = \frac{x^2}{(b/2)^2}$$

thus

$$(\tau_A - \tau_P)/\tau_A = (2x/b)^2 \tag{290}$$

or

$$\tau_P = \tau_A[1 - (2x/b)^2] \tag{291}$$

Similarly, using the parabola CDE to represent the stresses along the periphery CB, we have

$$\tau_Q = \tau_B[1 - (2y/h)^2] \tag{292}$$

It will be assumed that the stress component τ_x (Fig. 144*b*) along any line (such as WQ) parallel to AC varies in the same way as does the stress along AC, but that the value of τ_x decreases directly from the value τ_P on the surface at P to zero at R on the major axis; a similar

assumption is made with respect to τ_y. That is, if τ_x and τ_y denote the components of the stress at any point in the bar,

$$\tau_x = \frac{y}{h/2}\tau_P = \frac{2y}{h}\tau_A\left[1 - \left(\frac{2x}{b}\right)^2\right] \tag{293}$$

and

$$-\tau_y = \frac{x}{b/2}\tau_Q = \frac{2x}{b}\tau_B\left[1 - \left(\frac{2y}{h}\right)^2\right] \tag{294}$$

Further, it is assumed that τ_A and τ_B are inversely proportional to the distances of A and B from the center of the bar. That is,

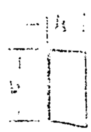
$$\tau_B = (h/b)\tau_A \tag{295}$$

These assumptions are in accord with the general behavior of the bar as indicated by Fig. 145a, although they must be regarded only as approximate since the exact manner of variation of stress cannot be determined in this way. By use of Eqs. 293, 294, and 295, the right-hand member of Eq. 289 may be expressed in terms of the maximum stress τ_A and the dimensions of the area. Thus,

$$T = \frac{2\tau_A}{h}\left(\int y^2\,da - \frac{4}{b^2}\int x^2y^2\,da\right) + \frac{2h}{b^2}\tau_A\left(\int x^2\,da - \frac{4}{h^2}\int x^2y^2\,da\right) \tag{296}$$

But

$$\int y^2\,da = I_x = \tfrac{1}{12}bh^3 \qquad \int x^2\,da = I_y = \tfrac{1}{12}hb^3 \qquad \int x^2y^2\,da = \frac{b^3h^3}{144} \tag{297}$$



Hence Eq. 296 becomes b > h

$$T = \tfrac{2}{9}bh^2\tau_A \quad \text{or} \quad \tau_A = \tfrac{9}{2}(T/bh^2) \tag{298}$$

in which τ_A is the maximum shearing stress in the bar when subjected to a twisting moment T, provided that h is not greater than b and that the stress does not exceed the shearing elastic strength of the material. If T is expressed in pound-inches and b and h in inches, τ_A will be expressed in pounds per square inch.

The above expression for the maximum stress must not be expected to give reliable results for a very thin section, that is, when b is very large compared to h. When b has a value of $1h$ to $2h$, the value of τ_A, though only approximate, may be considered to be reliable. If Eq. 298 is written

$$T = \alpha bh^2\tau_A \quad \text{or} \quad \tau_A = (1/\alpha)(T/bh^2) \tag{298a}$$

the values of α given in Table 13 will make the values of τ_A nearly the same as those given by St. Venant's analysis (see Appendix II).

From St. Venant's analysis the angle of twist ϕ per unit length is given by the equation

$$\phi = (1/\beta b h^3)(T/G) \tag{299}$$

where β has the values given in Table 13.

TABLE 13

$b/h = 1$	1.5	2	2.5	3	4	6	10	∞
$\alpha = 0.208$	0.231	0.246	0.256	0.267	0.282	0.299	0.312	0.333
$\beta = 0.141$	0.196	0.229	0.249	0.263	0.281	0.299	0.312	0.333

USED THIS IN CALCULATION

Problems

157. A steel bar having a rectangular cross section ½ in. wide and 1½ in. long is subjected to a twisting moment of 1000 lb-in. The shearing elastic strength of the material is 12,000 lb per sq in. (*a*) Calculate by Eq. 298 the maximum shearing stress, and state where in the bar it occurs. (*b*) Calculate the shearing stress at the center of the short side. *Ans.* (*a*) Max. τ = 12,000 lb/in.^{2}; (*b*) τ = 4000 lb/in.^{2}.

158. Solve Prob. 157 by making use of the values of α given in Table 13.
Ans. Max. τ = 10,000 lb/in.^{2}.

159. A bar of spring steel has a tensile elastic strength of 50,000 lb per sq in. and a shearing elastic strength equal to 0.6 of the tensile. The bar has a rectangular cross section and is subjected to a twisting moment of 5000 lb-in. If the working stress is two-thirds of the shearing elastic strength and the width of the section is ¾ in., what should be the length of the section?

160. A rectangular bar having a cross section such that $b/h = k$ and a cylindrical bar having a diameter d are subjected to the same twisting moment T. Using Eq. 298, show that when $d = 1.042h\sqrt[3]{k}$ the maximum shearing stresses in the two bars are equal, if the elastic strength of the material is not exceeded.

161. If the cross section of a rectangular bar is 1½ in. thick, what must be its width in order that the maximum shearing stress produced in it shall be the same as that in a cylindrical shaft 2 in. in diameter, both shafts being subjected to the same twisting moment? Use Eq. 298*a*.

162. Two bars, one with a square cross section and the other with a circular cross section, have equal cross-sectional areas and are subjected to equal twisting moments. Show by use of Eq. 298 that the maximum torsional shearing stress in the square bar is 1.27 times that in the circular shaft.

86 Elastic-membrane (soap-film) analogy. St. Venant in his analysis by the procedure discussed in Appendix II showed that, for a prismatic bar subjected to a twisting moment T, the shearing stresses on any plane cross section of the bar can be interpreted by the slopes of a dome-like surface over the cross section, such as is shown in Fig. 146. For example, suppose that this surface is represented for the section $ABCD$ of the bar by the surface whose cross sections in the yz and

xz planes are the arcs AEB and DEC, respectively; St. Venant showed that the slope of any line tangent to this surface at a point P, such as the line mn, is proportional to the shearing stress at the point P', the projection of P, in a direction perpendicular to the projection of the line mn in the cross section. Hence, if the Y axis is the projection of mn, the shearing stress, τ_x at P' in the x direction is proportional to the tangent of the angle that mn makes with the Y axis.

Furthermore, St. Venant showed that the volume beneath this surface is proportional to one-half of the twisting moment. The equation

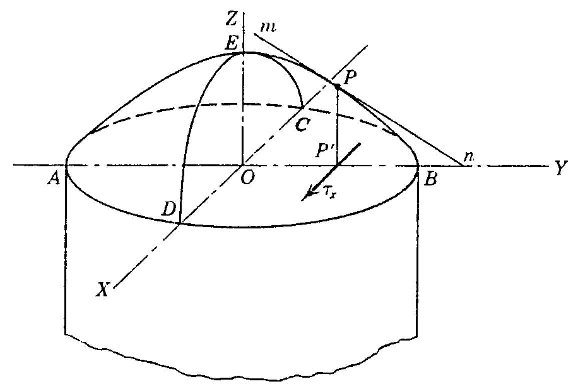


FIG. 146 Soap-film surface.

for this surface is called a *stress function.* Although St. Venant showed that such a surface (or stress function) always existed for a prismatic bar of any type of solid cross section, he actually found the stress function for only a relatively few sections such as ellipses, rectangles, equilateral triangles, etc. Weber has found the stress function for some other more irregular sections, but the mathematical equation for the stress function is very complicated for many of the technically important sections such as angle, channel, and I sections.

However, a close approximation to the surface representing the stress function for almost any shape of cross section can be made by the use of the soap-film analogy. A rather detailed description of the soap-film analogy is given in Appendix II. It was pointed out by Prandtl that the differential equation for the stress function of the twisted bar is of the same form and has the same boundary conditions as that of the surface of an elastic membrane stretched over an opening of the same shape as that of the cross section of the bar and distended by being subjected to a slight difference of pressure on its two sides. In 1917, A. A. Griffith and G. I. Taylor made use of this analogy for determining

the torsional strength and stiffness of airplane propeller blades and various structural shapes (see reference 3 for apparatus used). The desired shaped hole is cut in a thin metal plate, and a circular hole of a certain predetermined diameter is also cut in the same plate. The plate is then clamped between the two halves of a cast-iron box, and soap films are stretched across the two holes. Air is then forced into the lower compartment of the box, thereby causing the soap films to be distended into a dome-like surface by the uniform pressure. The following statements apply only to so-called solid areas and not to the cross sections of hollow bars such as hollow cylinders, hollow rectangles, etc. (thin-walled hollow sections are discussed in Art. 88):

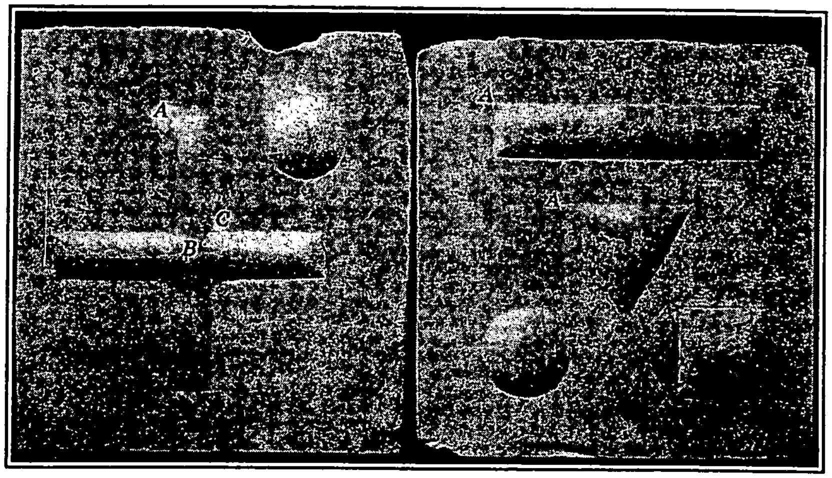
(*a*) The slope of a tangent line at any point on the soap film (see Fig. 146) is proportional to the shearing stress at the corresponding point in the bar in a direction at right angles to the projection in the section of the tangent line. The uniform pressure which causes the film to distend is proportional to the angle of twist; therefore, if soap films are stretched over two openings representing the cross sections of two bars and are subjected to the same uniform pressure, the stresses represented by the slopes of the soap films are stresses which would occur in the bars if they were subjected to the same angle of twist per unit of length.

Thus, if one of the two bars has a circular cross section, in which the stress at any point can be calculated satisfactorily, the stress at any point in a bar of irregular cross section may be found if the slopes to the films at corresponding points are determined experimentally.

(*b*) The volume under the soap film is proportional to the twisting moment acting on the bar. Thus, if the two bars are given the same angle of twist per unit of length, the twisting moment required to cause the given angle of twist of the irregular-shaped bar may be found if the ratio of the volumes under the two soap films is found experimentally.

Much of the importance of the soap-film analogy is based on the fact that, by visualizing the shape or form that the soap film will take, many valuable deductions may be made without the necessity of performing experiments. Figures 147 and 148 show views of the approximate forms of the soap films for several sections. These models were obtained by using a very thin sheet of rubber in place of the soap film and pouring plaster of Paris over the plate containing the holes, thereby causing the rubber membrane to deflect. The forms of these models are close approximations to the forms of the soap films for the same areas and will help the reader in visualizing the forms of the soap film for other areas. It will be noted that at outstanding corners, as at A, the slope to the soap film is practically zero, denoting zero stress; at inward projecting corners, as at C, the slope is very steep, denoting very high

stress; and along a long straight edge the slope is nearly constant. Further, it will be noted that at the intersection of two rectangular areas, as at B, the volume is increased as shown by the dome or hump, denoting increased twisting moment to produce a given angle of twist. The shadows on the models distort somewhat the true shape of the models; for example, the slopes at some of the outstanding corners do



FIGS. 147 and 148 Plaster models of soap films.

not appear to be zero, etc. The reader should carefully visualize the form of the soap films and verify the following statements:

I. *Stiffness.* Since the twisting moment required to produce a given angle of twist is proportional to the volume under the film, it is evident that:

(*a*) A torsional member having a long, thin, rectangular cross section is not as stiff as one having a square section of the same area.

(*b*) The torsional stiffness of a member with a long, straight, thin rectangular cross section is approximately the same as that of a member having an L-shaped or U-shaped section, etc., provided that the width of the section and the length of median line remain constant.

(*c*) Any cut in the section, such as a keyway in a shaft, reduces the stiffness of the shaft more than the cut reduces the area of the section.

(*d*) Any addition to the area of the given section increases its stiffness.

II. *Stress.* Since the stress at any point in a section for a given angle of twist is proportional to the slope of the film at the corresponding point on the film, it is evident that:

(*a*) The stress at a point on the boundary of the section where the section is convex outward (as at A, Fig. 149) is less than it would be if

the boundary of the section were straight; and at a sharp outwardly projecting corner the stress is zero.

(b) The stress at a point on the boundary of the section where the section has a concave curvature (as at B, Fig. 149) is greater than it would be if the section were straight; and the stress at an inward projecting sharp corner would be theoretically infinite if stress remained proportional to strain. However, if the material is ductile, it will deform

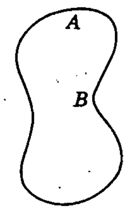


FIG. 149

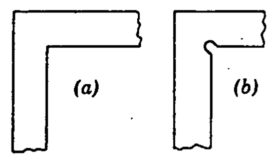


FIG. 150

and the stress will be distributed to the adjacent material. The stress at a re-entrant angle may be reduced by cutting away some of the material in the corner, as indicated in Fig. 150.

(c) The maximum shearing stress in a section occurs at or near one of the points of contact of the largest inscribed circle, not, in general, at the point on the boundary nearest the centroid of the section, as has sometimes been assumed. If, however, the boundary is more concave at some other point, the maximum stress may occur at the point of greater concavity. For example, in a rectangle (Fig. 151a) the slope

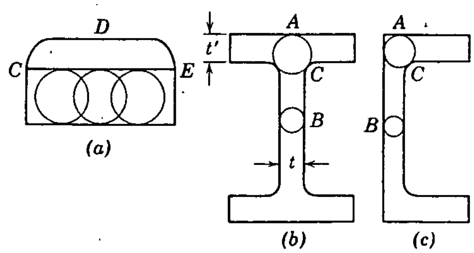
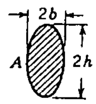
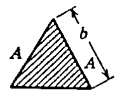
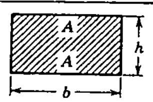
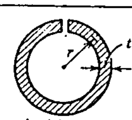
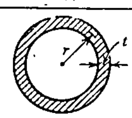
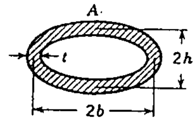
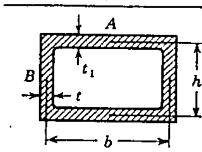


FIG. 151

to the film would be approximately constant along the greater part of the long side CE, and hence the shearing stress is nearly constant along this side. Therefore, the shearing stress probably varies about as indicated by the ordinates to the curve CDE rather than by the ordinates to a parabola as was assumed in the method of solution in Art. 85. For the I section and the channel section (Figs. 151b and 151c) with the

TABLE 14

Approximate Formulas for Torsional Shearing Stress and Angle of Twist, Obtained from Mathematical Analysis

Cross Section	Relation between Shearing Stress and Twisting Moment	Relation between Angle of Twist per Unit Length and Twisting Moment
 Fig. A	$\tau_A = \dfrac{2T}{ab} = \dfrac{2T}{\pi h b^2}$	$\phi = 4\pi^2 \dfrac{J}{a^4} \cdot \dfrac{T}{G} = \dfrac{h^2 + b^2}{\pi h^3 b^3} \cdot \dfrac{T}{G}$
 Equilateral triangle Fig. B	$\tau_A = \dfrac{20T}{b^3}$	$\phi = \dfrac{80}{b^4\sqrt{3}} \cdot \dfrac{T}{G} = \dfrac{46.2}{b^4} \cdot \dfrac{T}{G}$
 Fig. C	$\tau_A = \dfrac{T}{\alpha b h^2}$	$\phi = \dfrac{1}{\beta b h^3} \cdot \dfrac{T}{G}$
	For values of α and β, see Table 13.	
 Equivalent to rectangle with $\frac{b}{h}$ = large Fig. D	$\tau = \dfrac{3T}{2\pi r t^2} = \dfrac{3r}{t} \cdot \dfrac{T}{2\pi r^2 t}$	$\phi = \dfrac{3}{2\pi r t^3} \cdot \dfrac{T}{G} = \dfrac{3r^2}{t^2} \cdot \dfrac{1}{2\pi r^3 t} \cdot \dfrac{T}{G}$
 Fig. E	$\tau = \dfrac{Tr}{J} = \dfrac{T}{2\pi r^2 t}$	$\phi = \dfrac{T}{JG} = \dfrac{1}{2\pi r^3 t} \cdot \dfrac{T}{G}$
 Fig. F	$\tau_A = \dfrac{T}{2\pi b h t}$	$\phi = \dfrac{\sqrt{2(b^2 + h^2)}}{4\pi b^2 h^2 t} \cdot \dfrac{T}{G}$
 Fig. G	$\tau_A = \dfrac{T}{2bht_1}$, $\tau_B = \dfrac{T}{2bht}$	$\phi = \dfrac{bt + ht_1}{2tt_1 b^2 h^2} \cdot \dfrac{T}{G}$

T = twisting moment. ϕ = angle of twist per unit of length. a = area of cross section. J = polar moment of inertia. G = shearing modulus of elasticity.

thickness of the flange greater than that of the web, the slope of the film (and hence the stress) would be greater at A than at B; but the slope at C would be greater than at either A or B.

(*d*) For a given angle of twist the stress in a bar at the points of contact of an inscribed circle is always greater than the stress in a circular bar whose radius is equal to that of the inscribed circle. The more nearly the boundary of the section coincides with the arc of the circle at the point of contact, the nearer the stress at the point approaches that in the inscribed circular bar.

87 Special formulas obtained from mathematical analysis for solid cross sections. As noted in the preceding article, a mathematical analysis of the torsion of a bar has been made by St. Venant only for the simpler cross sections, and the results even for most of these sections are too complicated for convenient use. In Appendix II St. Venant's method of analysis is discussed and illustrative problems are given which show how his method is applied. Stresses for bars having the first four cross sections shown in Table 14 were obtained by St. Venant's analysis for bars having solid cross sections. The maximum shearing stress τ in each case occurs along the central portion of the longest side of the cross section, as at A in the figures of Table 14. By visualizing the slopes to the soap films for the so-called *solid* areas in Table 14 the point or region of maximum shearing stress can easily be determined. In the expressions for the angle of twist per unit length in the third column of Table 14 the ratio T/G is multiplied by a constant which depends upon the shape and dimensions of the cross-sectional area. This constant, sometimes called the torsion constant, is a measure of the torsional rigidity and corresponds to the polar moment of inertia J in Eq. 288 which applies to a circular section. The method of obtaining the stresses and angle of twist in the *hollow thin-walled* tubes included in Table 14 is given in the next paragraph.

Problems

163. Solve Prob. 157 by use of the equations in Table 14. Also calculate the angle of twist of the bar described in Prob. 157, assuming its length to be 4 ft.

164. A steel bar 2 in. by 2 in. by 6 ft long is subjected to twisting moments at its ends that cause a maximum shearing stress equal to one-half of the shearing elastic strength of the material. Calculate the angle of twist of one end of the bar relative to the other. Assume the shearing elastic strength of the steel to be 30,000 lb per sq in.

165. A steel bar having a slender rectangular cross section ($\frac{1}{4}$ in. by 4 in.) is subjected to twisting couples of 1000 lb-in. at its ends. Find the maximum shearing stress and the angle of twist per unit length.